ANOMALY-FREE TWO-DIMENSIONAL CHIRAL SUPERGRAVITY–MATTER MODELS AND CONSISTENT STRING THEORIES

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We discuss the cancellation of Weyl and local Lorentz anomalies in two-dimensional chiral $N \leqslant 2$ supergravity–matter systems. There exists a unique anomaly-free model with $N = \frac{1}{2}$ supersymmetry which corresponds to the heterotic string (with the critical dimension $D = 10$) in the same way as the anomaly-free models with non-chiral $N = 0$, 1 and 2 supersymmetries correspond to the string theories with $D = 26$, 10 and 2. The only two other anomaly-free models have chiral $N = 1_{++}$ and $N = \frac{3}{2}{}_{++-}$ supersymmetries and correspond to two new "heterotic" string theories with $D = 2$.

1. It was known for some time that there exist only three consistent geometrical string theories (with critical dimensions $D = 26, 10$ and 2) based on $N = 0$, 1 and 2 supersymmetric two-dimensional actions. The values of the critical dimension can be derived [1–3] from the condition of cancellation of super-Weyl anomalies in the corresponding [4,5] $d = 2$ supergravity–matter systems. Here N is the number of supersymmetry generators considered as $d = 2$ Majorana spinors. However, the simplest ("$N = \frac{1}{2}$") supersymmetry algebra in two dimensions has a Majorana–Weyl generator (see e.g. refs. [6–8]). Thus in addition to non-chiral ($N = 1_{+-}, N = 2_{++--}$) $d = 2$ supergravities there is a number of chiral ones [‡1]:

$$N = \tfrac{1}{2}{}_{+}, \quad N = 1_{++}, \quad N = \tfrac{3}{2}{}_{+++},$$
$$N = \tfrac{3}{2}{}_{++-}, \quad N = 2_{++++}, \quad N = 2_{+++-}, \tag{1}$$

(plus their chiral conjugates).

The recent discovery of a new heterotic string theory [9] (originally constructed as a chiral combination of the $D = 26$ and the $D = 10$ string theories) which can be viewed as based on $N = \frac{1}{2}$ $d = 2$ supersymmetry suggests a systematic study of anomaly cancellation in $N \leqslant 2$, $d = 2$ chiral supergravity–matter models. Anomaly-free models may correspond to new consistent string theories [‡2].

We shall find below that there exist only three (Weyl- and Lorentz-) anomaly-free $d = 2$ theories based on $N = \frac{1}{2}{}_{+}$, $N = 1_{++}$ and $N = \frac{3}{2}{}_{++-}$. The first theory corresponds to the heterotic string [9] with the critical dimension $D = 10$ while the second and the third correspond to new string theories with $D = 2$ and can be interpreted as "chiral combinations" of the "non-chiral" $D = 2$ and $D = 26$ and $D = 2$ and $D = 10$ string theories (the existence of these two theories was already suggested in ref. [9]) [‡3].

The sequence of the numbers of supersymmetry generators of anomaly-free $d = 2$ supergravity models and of the critical dimensions of the corresponding string theories ($N = 0, D = 26; N = 1, D = 10; N = 2, D = 2$) suggests that an $N = \frac{1}{2}$ theory may correspond

‡1 We shall consider only theories with $N \leqslant 2$. It was noted in ref. [3] that string theories based on $N > 2$ $d = 2$ supergravities are unlikely to be consistent (they have negative "critical dimensions", i.e. it is impossible to cancel the Weyl anomalies).

‡2 By consistency of a string theory here we understand only the absence of $d = 2$ anomalies. We ignore the "next-level" questions of boundary conditions and truncation of the spectrum (e.g. we do not discriminate between the original spinning string and Green–Schwarz superstring).

‡3 Symbolically, the chiral half of $N = 2_{++--}$ theory is, e.g., $N = 1_{++}$ theory which is to be combined with the opposite chirality half of either the $N = 0$ theory or of the $N = 1_{+-}$ theory (i.e. the $N = \frac{1}{2}{}_{-}$ theory).

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to a string theory with $D = 18$ (spaces of dimensions $D = 2, 10, 18, 26, \ldots$ have a number of common features, e.g., admit Majorana–Weyl spinors, see also ref. [10]). We shall see that there is no anomaly-free theory which can be interpreted as a string theory in $D = 18$. It is the heterotic string (in $D = 10$) that corresponds to the unique anomaly-free $d = 2$ theory with the $N = \frac{1}{2}$ supersymmetry.

2. We shall start with recalling the structure of the non-chiral $N = 2, d = 2$ supergravity [5]. The gauge fields are a zweibein e_m^a, two Majorana gravitinos χ_m^A and a vector A_m ‡4. The gauge symmetries are the general coördinate invariance (with parameter ξ^m), the local Lorentz symmetry ($\delta\chi_m = -\frac{1}{2} l\rho_3\chi_m$), the Weyl symmetry ($\delta e_m^a = \lambda e_m^a$), the ordinary and conformal supersymmetries ($\delta\chi_m^A = \mathcal{D}_m\epsilon^A + \rho_m\zeta^A$) and the "phase" and "chiral" gauge invariances ($\delta\chi_m = \mathrm{i}(\alpha - \rho_3\beta)\chi_m$, $\delta A_m = \partial_m\alpha + \epsilon_{mn}\partial^n\beta$). Thus the gauge fields can be completely gauged away at the classical level. Assuming that a regularization prescription respects the general coordinate invariance we can group the anomalies in the two multiplets: "super-Weyl" (SW) (λ, ζ^A, β anomalies) and "super-Lorentz" (SL) (l, ϵ, α anomalies) ‡5. The SL (local Lorentz, ordinary supersymmetry and phase) anomalies automatically cancel in non-chiral theories. To cancel the SW-anomalies one is to couple the $N = 2$ supergravity to a number ($D = 2$ [3]) of $N = 2$ scalar multiplets $(x^\mu, y^\mu, \psi^{\mu A})$ ‡6.

We shall assume that the $N \leqslant 2$ chiral supergravities (1) can be obtained (e.g. by chiral truncations) from the non-chiral $N = 2_{++--}$ theory. Then the SL-anomalies also may not a priori cancel ‡7. Here we shall study only the cancellation of the Weyl and Lorentz gravitational anomalies (their cancellation implies cancellation of all other SW and SL anomalies if a classical theory is supersymmetric so that the anomalies form the supermultiplets). In $d = 2$ the cancellation of the W and L anomalies is equivalent to the vanishing of the effective action computed on a gravitational (e_m^a) background.

The finite parts of the effective actions for a real scalar and a Majorana spinor are (see e.g. ref. [1])

$$\Gamma_0 = \tfrac{1}{2}\ln\det\Delta_0 = \frac{1}{96\pi}\int R\Box^{-1}R, \quad \Delta_0 = -\nabla_m\nabla^m,$$

$$\int R\Box^{-1}R = -\int \mathrm{d}^2z\,\mathrm{d}^2z'(\sqrt{g}R)_z\Delta_0^{-1}(z,z')(\sqrt{g}R)_{z'}, \tag{2}$$

$$\Gamma_{1/2} = -\tfrac{1}{2}\ln\det(\mathrm{i}\rho^m\mathcal{D}_m)_\mu = -\tfrac{1}{4}\ln\det\Delta_{1/2} = \tfrac{1}{2}\Gamma_0,$$

$$\mathcal{D}_m = \partial_m + \mathrm{i}\omega_m\rho_3, \quad \omega_m = -\tfrac{1}{4}\epsilon^{ab}(\partial_n e_m^b)e_a^n,$$

$$\Delta_{1/2} = -\mathcal{D}^2 + R/4. \tag{3}$$

The effective actions for the graviton and the Majorana gravitino are completely given by the general coordinate and ϵ-supersymmetry ghost contributions corresponding to the gauges $g_{mn} = \mathrm{e}^{2\rho}\delta_{mn}, \chi_m = \rho_m\chi$ [1–3]:

$$\Gamma_2 = -\tfrac{1}{2}\ln\det\Delta_1 = -26\Gamma_0,$$

$$\Delta_{1mn} = -\nabla_{mn}^2 - \tfrac{1}{2}Rg_{mn}, \tag{4}$$

$$\Gamma_{3/2} = 2\ln\det(\mathrm{i}\rho^{m\mathrm{T}}\mathcal{D}_m)_\mathrm{M} = \ln\det\tilde{\Delta}_{1/2} = 11\Gamma_0,$$

$$\tilde{\Delta}_{1/2} = -\mathcal{D}^2 - R/4. \tag{5}$$

The effective action for an (anti)self-dual antisymmetric tensor ("chiral scalar") and a Majorana–Weyl spinor or gravitino gets an imaginary contribution due the local Lorentz anomaly (cf. refs. [11,12]):

$$\Gamma_{0\pm} = \tfrac{1}{2}\ln\det(\partial_m \pm \mathrm{i}\epsilon_{mn}\partial^n)$$

$$= \frac{1}{192\pi}\int R\Box^{-1}(R \mp \mathrm{i}\nabla_m\omega^m), \tag{6}$$

$$\Gamma_{1/2\pm} = -\tfrac{1}{4}\ln\det(\mathrm{i}\rho^m\mathcal{D}_m)_\pm = \tfrac{1}{2}\Gamma_{0\pm}, \tag{7}$$

$$\Gamma_{3/2\pm} = 2\times\tfrac{1}{2}\ln\det(\mathrm{i}\rho^{m\mathrm{T}}\mathcal{D}_m)_\pm = 11\Gamma_{0\mp}. \tag{8}$$

Here ω_m is the Lorentz connection and $\rho_3\psi_\pm = \pm\psi_\pm$. Note that $\Gamma_s = \Gamma_{s+} + \Gamma_{s-}, s = 0, \frac{1}{2}, \frac{3}{2}$. The total results for the coefficients of the Weyl and Lorentz

‡4 Here $a, m = 1, 2$, $A = 1, 2$. ρ^a will denote the two-dimensional Dirac matrices, $\rho_3 = \sigma_3$. Throughout the paper we ignore auxiliary fields.

‡5 The corresponding $N = 1$ and $N = \frac{1}{2}$ multiplets of anomalies are (λ, ζ) and (l, ϵ).

‡6 Here $\mu = 1, \ldots, D$; x and y are real scalars and ψ^A are two Majorana spinors. Note that in the string interpretation x^μ are space–time coordinates while y^μ are internal "charge" variables. One cannot identify y^μ with space coordinates because the corresponding $N = 2$ supersymmetric action [5] is not invariant under the O(2D)-Lorentz group.

‡7 The local Lorentz anomaly in $d = 2$ was first computed in ref. [11] (see also ref. [12]). The local supersymmetry anomaly for the $N = \frac{1}{2}, d = 2$ scalar multiplet was studied in ref. [8]. The abelian phase anomaly in $d = 2$ was discussed, e.g., in refs. [13,12].

anomaly (i.e. the coefficients of the real and imaginary parts of the effective action) in a theory containing one graviton and chiral scalars, spinors and gravitinos are

$$W = \tfrac{1}{2}(n_{0+} + n_{0-}) + \tfrac{1}{4}(n_{1/2+} + n_{1/2-}) + \tfrac{11}{2}(n_{3/2+} + n_{3/2-}) - 26 , \quad (9)$$

$$L = n_{0+} - n_{0-} + \tfrac{1}{2}(n_{1/2+} - n_{1/2-}) - 11(n_{3/2+} - n_{3/2-}) . \quad (10)$$

Note that (as follows from (6)–(8)) the coefficient of the Lorentz anomaly for $s = 0, \frac{1}{2}, \frac{3}{2}$-fields is proportional to the coefficient of the Weyl anomaly.

3. Now let us analyze the conditions $W = 0, L = 0$ starting with the $N = 0$ supersymmetry case. As we can always trade a chiral scalar for two Majorana–Weyl spinors, let us take $n_{0+} = n_{0-} \equiv D$. Then $L = 0$ implies $n_{1/2+} = n_{1/2-} \equiv n$ and $W = D + n/2 - 26 = 0$. For $n = 0$ we get the standard Bose string theory, while $n = 2r \neq 0$ corresponds to the Bardakci–Halpern string [14].

Next let us consider the case of $N = \frac{1}{2}$ supersymmetry. We can couple the $N = \frac{1}{2}_+$ supergravity multiplet (e_m^a, χ_{m+}) to D $N = \frac{1}{2}_+$ matter multiplets (x^μ, ψ_-^μ) [8] and to a number of *positive* chirality spinors (introduction of additional negative chirality spinors breaks down the local $N = \frac{1}{2}_+$ supersymmetry) [‡8]. An even number of positive chirality spinors can of course be replaced by the corresponding number of chiral scalars $(n_{0+} = \frac{1}{2} n_{1/2+})$. For definiteness we always take the "singlet" positive chirality fields to be spinors. Then (9) and (10) imply

$$W = D + \tfrac{1}{4}D + \tfrac{1}{4} n_{1/2+} + \tfrac{11}{2} - 26 = 0,$$

$$L = \tfrac{1}{2} n_{1/2+} - \tfrac{1}{2}D - 11 = 0 . \quad (11)$$

The unique solution of (11) is $D = 10, n_{1/2+} = 32$ which corresponds to the heterotic string theory [9] [‡9]. The contribution to W of the 32 Majorana–Weyl spinors is the same as that of 8 real (non-chiral) scalars. If we consider these scalars as additional space coordinates the resulting theory may look as corresponding to a hypothetical "$D = 18$ string". However the resulting theory is not free of local Lorentz (and supersymmetry) anomalies (introducing "singlet" real scalars we also break local $N = \frac{1}{2}_+$ supersymmetry at the classical level). Thus it seems to be no consistent "$D = 18$ string", the heterotic string being the only string theory based on an anomaly-free $N = \frac{1}{2}$ supersymmetric two-dimensional model.

The $N = 1_{++}$ supergravity is a chiral truncation of the $N = 2_{++--}$ theory: $(e_m^a, \chi_{m+}^A, A_{m+} = \epsilon_{mn} A_+^n)$. In addition to the contributions of the graviton and gravitinos $(n_{3/2+} = 2)$ the supergravity effective action contains also the contribution of one chiral scalar ghost corresponding to the gauge $A_{m+} = 0$ (cf. ref. [3]), i.e. effectively we have $n_{0-} = -2$. Coupling the theory to D ("reducible") $N = 1_{++}$ matter multiplets $(x^\mu, y_-^\mu, \psi_+^{\mu A})$ and a number of "singlet" positive chirality spinors we get

$$W = (D + \tfrac{1}{2}D - 1) + \tfrac{1}{2}D + \tfrac{1}{4} n_{1/2+} + 11 - 26 = 0 ,$$

$$L = 2 - D + \tfrac{1}{2} n_{1/2+} - D - 22 = 0 , \quad (12)$$

and thus the anomalies cancel for $D = 2, n_{1/2+} = 48$. Introducing instead the 48 Majorana–Weyl spinors 24 chiral scalars we can identify the resulting theory with a "chiral combination" of the $N = 2, D = 2$ and the $N = 0, D = 26$ string theories.

A similar analysis can be carried out for the $N = \frac{3}{2}_{++-}$ case. The supergravity contribution here is the same as for the $N = 1_{++}$ plus $N = \frac{1}{2}_-$ theory (with the graviton contribution counted once). Introducing D matter multiplets $(x^\mu, y_-^\mu, \psi_+^{\mu A}, \psi_-^\mu)$ and $n_{1/2+}$ additional chiral spinors we get a unique anomaly-free solution: $D = 2, n_{1/2+} = 24$. The corresponding string theory can be again considered as a "chiral combination" of the non-chiral $N = 2, D = 2$ and $N = 1, D = 10$ string theories.

The remaining chiral theories with $N = \frac{3}{2}_{+++}, N = 2_{++++}$ and $N = 2_{+++-}$ supersymmetry turn out to have either Lorentz or Weyl anomalies. For the $N = 2$ theories this is easy to understand observing that chirality does not matter for the Weyl anomaly and that $W = 3(D - 2)$ for the $N = 2$ supergravity coupled to D $N = 2$ matter multiplets [3]. To cancel the Lorentz anomaly we are to introduce additional

‡8 We use a "reducible" [7] $N = \frac{1}{2}_+$ scalar multiplet: (x_+, x_-, ψ_-) (x_+ is singlet under the $N = \frac{1}{2}_+$ supersymmetry). Note that the chirality of the spinor ψ is opposite to that of the gravitino (i.e. to that of the supersymmetry parameter).

‡9 The cancellation of $d = 2$ Weyl and Lorentz anomalies in the heterotic string theory was also checked in ref. [9].

Majorana–Weyl spinors, but they give positive contribution to W making it non-vanishing even for $D < 2$. The same conclusion remains true for the $N = \frac{3}{2}_{+++}$ case.

4. We finish with several remarks. We have studied the cancellation of W- and L-anomalies which are "representatives" of the two anomaly supermultiplets. It would be interesting also to carry out an explicit check of cancellation of supersymmetry anomalies in the three W- and L-anomaly free models we found. If the Lorentz and supersymmetry anomalies belong to one multiplet [‡10] the additional positive chirality spinors we introduced to provide $W = L = 0$ should also contribute to the supersymmetry anomaly. Hence it remains to be proved that, e.g., the $D = 10$ heterotic string theory is free of $d = 2$ supersymmetry anomalies [‡11]. An analogous remark can be made concerning the problem of the chiral gauge anomaly cancellation in $N = 1_{++}$ and $N = 3_{++-}$ theories.

Our starting point was the condition of cancellation of the Weyl and Lorentz anomalies. However, it was suggested in refs. [1,2] that relaxing the SW-anomaly cancellation constraints and introducing at the same time the corresponding "anomalous" degrees of freedom as additional variables one may define the $N = 0$ and $N = 1$ string theories below the critical dimensions ($D = 26$ and $D = 10$). Here we would like to note that an analogous suggestion (to relax the condition of cancellation of SL-anomalies) can be made also for string theories based on chiral $d = 2$ supersymmetries. Consider the Lorentz and Weyl anomaly case using the following parametrization (general coordinate gauge) of the zweibein [12]: $e_a^m = e^{-\rho}(\delta_{am} \cos\sigma - \epsilon_{am} \sin\sigma), g_{mn} = e^{2\rho}\delta_{mn}$ (ρ and σ are W- and L-anomalous degrees of freedom). Then [see (6)–(8)]

$$\int R\Box^{-1}(R - i\nabla_m \omega^m) = -4 \int d^2z\, \partial_m\rho(\partial_m\rho + i\partial_m\sigma). \tag{13}$$

Thus even if the Weyl anomaly ($\sim \int R\Box^{-1}R$) "cancels out" the euclidean effective action contains an imaginary piece ($\sim i\int \partial\rho\, \partial\sigma$) corresponding to two scalar degrees of freedom (the absence of ghosts depends on the sign of the total Lorentz anomaly). This suggests a possibility to have a unitary chiral $d = 2$ gravitational theory with a non-vanishing Lorentz anomaly (for an analogous proposal in the case of the chiral $d = 2$ gauge theory see ref. [13]).

‡10 This should be as the classical theory is locally supersymmetric. Note that it is the local supersymmetry of the chiral anomaly free theories we discussed that differs them from the Bardakci–Halpern models, where both negative and positive chirality spinors can be introduced in an arbitrary way.

‡11 This point is somewhat mysterious given that no couplings of the positive chirality gravitino to the positive chirality spinor survive after truncation of the $N = 1_{+-}$ supergravity–matter system to a $N = \frac{1}{2}_{+}$ system.

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