

Supersymmetric version of the 3D Virasoro-like algebra and supersymmetrization of $SU(\infty)$

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Extended supersymmetric versions of the Virasoro-like generalization of a 3D conformal algebra constructed previously by us are obtained. They contain supersymmetric versions of $SU(\infty)$ as sub-superalgebras.

1. Introduction

In the present paper we continue our investigation of the possible infinite-dimensional analogues of the Virasoro algebra in $D > 2$ space-time initiated in refs. [1,2]. Using the symplectic structure on the twistor space, in ref. [1] we have constructed a certain infinite-dimensional algebra which contains a 3D conformal algebra ($so(3, 2)$ or $so(4, 1)$) as its finite-dimensional subalgebra. It has a structure somewhat like the Virasoro one. Our goal in this paper consists in constructing N -extended supersymmetric versions of the algebra $PAC(so(3, 2))$ of ref. [1]. Such superalgebras may play the role of straightforward algebraical counterparts of the Neveu-Schwarz and $SO(N)$ -extended [3] conformal superalgebras in $D=3$. They contain $osp(N|4)$ as their finite-dimensional sub-superalgebras. Higher spin generalizations of these superalgebras looking like $SO(N)$ -extended higher spin conformal superalgebras in $D=2$ [4,5] are also obtained. In the course of our consideration we shall also construct N -extended supersymmetric versions of $SU(\infty)$ in a rather simple way^{*1}. Our method will be based on the Poisson superbrackets on the extended supertwistor superspace.

We will follow the notations and conventions of ref. [1] and sometimes refer to the formulae therein as e.g. (I.3).

2. $SO(N)$ -extended 2D higher spin conformal superalgebras

To begin with, let us consider the simpler case of the $SO(N)$ -extended 2D conformal superalgebra [3] and its higher spin version [4,5].

First, let us introduce a pair of canonical variables (p, q) and N Grassmann variables $\psi_i, i=1, \dots, N$, and define the Poisson superbrackets as usual,

$$[f, g]_{PB} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q} + 2 \frac{\partial_r f}{\partial \psi_i} \frac{\partial_l g}{\partial \psi^i}. \quad (1)$$

We will be concerned with a Poisson superbracket algebra of the Laurent polynomials of p and q and usual polynomials of ψ_i generated by the basis

^{*1} In this paper $su(\infty)$ is just the Poisson bracket (area-preserving diffeomorphisms) algebra on the sphere S^2 [12].

$$L_{n,i_1 \dots i_k}^s = \frac{1}{2} (p)^{s+n} (q)^{s-n} \psi_{i_1} \dots \psi_{i_k}, \tag{2}$$

where $k=0, 1, \dots, N$; $s \in \mathbb{Z}$ and $n \in \mathbb{Z}$, or $s \in \mathbb{Z} + \frac{1}{2}$ and $n \in \mathbb{Z} + \frac{1}{2}$ (we consider the Neveu-Schwarz sector) and the following conditions are supposed to be satisfied: $k+2s \geq 2$ and $k+2s$ is even. The Grassmann parity for such functions is defined as usual: $L_{n,i_1 \dots i_k}^s$ is Grassmann even when s is integer and k is even, and it is odd when s is half-integer and k is odd (note that we require $k+2s$ to be even for the right connection between spin and statistics for (2)). Calculating the Poisson brackets (1) for the basis elements (2), we obtain the composition law in the superalgebra, which we denoted $\text{shsc}^{*E}(N|1)$ in refs. [4,5] (shsc means super higher spin conformal, E (even) for $k+2s$ even, * for the Poisson brackets)

$$\{L_{n,i_1 \dots i_k}^s, L_{n',j_1 \dots j_k}^{s'}\}_{\text{PB}} = (sn' - s'n) L_{n+n',i_1 \dots i_k j_1 \dots j_k}^{s+s'-1} + kk' \text{Alt}(\delta_{ij} L_{n+n',i_1 \dots i_{k-1} j_2 \dots j_k}^{s+s'}), \tag{3}$$

where Alt means antisymmetrization in all i - and j -indices separately ($\text{Alt}(A_{i_1} B_{i_2}) = \frac{1}{2}(A_{i_1} B_{i_2} - A_{i_2} B_{i_1})$). The superalgebra $\text{shsc}^{*E}(N|1)$ (3) contains the $\text{SO}(N)$ -extended conformal superalgebra [3] as a subsuperalgebra generated by the monomials (2) with $2s+k=2$ (only spins $s \leq 1$ may appear)

$$L_n^1, L_{n,i}^{1/2}, \dots, L_{n,i_1 \dots i_N}^{-N/2} \quad (k+2s=2). \tag{4}$$

When $N=0$ this is the usual (centreless) Virasoro algebra and when $N=1$ it is the Neveu-Schwarz superalgebra. An important feature of the $\text{SO}(N)$ -superconformal algebra and its higher spin generalization (3) consists in the presence of negative spins ($s < 0$) when $N > 2$. Also when $N > 4$ [6] there is no non-trivial central extension. In refs. [4,5] we also calculated the conformal anomalies (from the nilpotency condition $\Omega^2=0$) for these extended higher spin conformal superalgebras and the results were $C = -\frac{1}{6}(N=0)$, $-\frac{1}{24}(N=1)$, $-\frac{1}{6}(N=2)$, and 0 (for all $N > 2$). To obtain the first three non-zero anomalies the ζ -regularization is needed, while the conformal anomalies for $N > 2$ are cancelled out separately for each $\text{osp}(N|2)$ supermultiplet ^{a2}.

Another interesting subsuperalgebra of (3) is extracted when the restrictions $s \geq 0$ and $|n| \leq s$ are supposed to be satisfied (i.e. the generators contain only non-negative powers of p and q). In fact this is a higher spin superalgebra $\text{shs}^{*E}(N|2)$ of ref. [7], where the extended versions of the higher spin algebras originally appeared in ref. [8] were constructed. It generalizes $\text{osp}(N|2)$ which is contained in $\text{shs}^{*E}(N|1)$ as a maximal finite-dimensional subsuperalgebra generated by $\{L_{\pm 1,0}^1, L_{\pm 1/2,j}^{1/2}, L_{0,j}^0\}$. On the other hand, in the terminology of refs. [1,2], $\text{shsc}^{*E}(N|1)$ is an analytic continuation of $\text{shs}^{*E}(N|1)$ and is obtained from the latter after abolishing the restrictions $|n| \leq s, s \geq 0$. (In such a terminology the $\text{SO}(N)$ -extended conformal superalgebra [3] is an analytic continuation of $\text{osp}(N|2)$ along the $\text{sp}(2)$ roots [2].)

3. Virasoro-like extension of 3D conformal superalgebra $\text{osp}(N|4)$ and its higher spin version

Now let us pass to the main subject of the present paper, the 3D conformal superalgebra. Along with the spinorial generating elements a_α, b_α as in (1.9) we introduce also Grassmann variables $\psi_i, i=1, \dots, N$ to obtain an extended supersymmetric version of the results of ref. [1]. (This is a general method of constructing extended higher spin superalgebras [8-10]. For finite-dimensional superalgebras such oscillator-like realizations were studied in ref. [11].) The Poisson superbracket for two functions of a_α, b_α and ψ_i is defined according to

$$[f, g]_{\text{PB}} = f \overset{\leftrightarrow}{\Delta} g, \quad \overset{\leftrightarrow}{\Delta} = \frac{\overset{\leftrightarrow}{\partial}}{\partial a_\alpha} \epsilon_{\alpha\beta} \frac{\overset{\leftrightarrow}{\partial}}{\partial b_\beta} + \frac{\overset{\leftrightarrow}{\partial}}{\partial b_\alpha} \epsilon_{\alpha\beta} \frac{\overset{\leftrightarrow}{\partial}}{\partial a_\beta} + 2 \frac{\overset{\leftrightarrow}{\partial}_i}{\partial \psi_i} \frac{\overset{\leftrightarrow}{\partial}_j}{\partial \psi_j}. \tag{5}$$

An algebra of all second order polynomials of $(a_\alpha, b_\alpha, \psi_i)$ with respect to the Poisson bracket (5) is iso-

^{a2} For comparison we give the conformal anomalies for usual conformal superalgebras without higher spins: $\frac{1}{6}(N=0)$, $\frac{1}{24}(N=1)$, $\frac{1}{6}(N=2)$ and 0 (for all $N > 2$).

morphic to $osp(N|4)$. An infinite-dimensional algebra $shsc^*(N|3)$ [9] of all order polynomials, so that

$$P(-a_\alpha, -b_\alpha, -\psi_i) = P(a_\alpha, b_\alpha, \psi_i), \tag{6}$$

generalizes the 3D conformal superalgebra $osp(N|4)$ to the case of all higher spins. $shsc^*(N|3)$ (N -extended higher spin 3D conformal Poisson-bracket superalgebra) is isomorphic to $shs^{*E}(N|4)$ [8] (higher-spin superalgebra in 4D anti-de Sitter space) due to the isomorphism between the usual 3D conformal and 4D anti-de Sitter superalgebras.

The generators of $shsc^*(N|3)$ have a form [9] (for all our notations and conventions see section 4 in ref. [1])

$$T_{\alpha(2\ell), i_1 \dots i_k}^{(s,c)} = \frac{1}{2} \sqrt{\frac{(2\ell)!}{(\ell+c)!(\ell-c)!}} \underbrace{a_{\alpha} \dots a_{\alpha}}_{\ell-c} \underbrace{b_{\alpha} \dots b_{\alpha}}_{\ell+c} \psi_{i_1} \dots \psi_{i_k} (a^\gamma b_\gamma)^{s-\ell}, \tag{7}$$

where the indices have the following range of definition:

$$s=0, \frac{1}{2}, 1, \dots, \quad \ell=0, \frac{1}{2}, 1, \dots, \quad \ell \leq s, \tag{8}$$

$c = -\ell, -\ell+1, \dots, \ell; s, \ell$ and c are simultaneously integer or half-integer; $k=0, 1, \dots, N, k+2s \geq 2$, and $k+2s$ is an even number.

The new element here in comparison with (I.12) consists in introducing the Grassmann variables ψ_i (each generator now is an anti-symmetric tensor with respect to its internal indices), and now s, ℓ and c may be integer and half-integer (Bose and Fermi generators are presented).

The second-order monomials (7) with $k+2s=2$ form a maximal finite-dimensional subalgebra isomorphic to $osp(N|4)$.

The superalgebra $shsc^*(N|3)$ is a 3D analogue of the *little* higher spin conformal subsuperalgebra of $shsc^*(N|1)$ given in (3) (note that "little" means that the restrictions $s \geq 0, |n| \leq s$ are supposed to be satisfied).

To pass to the 3D counterpart of the $SO(N)$ -extended Virasoro superalgebra and its higher spin version, one has to abolish the restrictions

$$s \geq 0, \quad \ell \leq s \tag{9}$$

in (7), (8) (all the others remain true). This is similar to abolishing the restrictions $s \geq 0, |n| \leq s$ that has to be done to pass to the Virasoro superalgebra from the little superalgebra. Such a general procedure was named analytic continuation in refs. [2,1].

Calculating the Poisson brackets of two basis monomials (7), we obtain a manifest expression for the N -extended supersymmetric version of the algebra (I.13):

$$\begin{aligned} & [T_{\alpha(2\ell), i_1 \dots i_k}^{(s,c)}, T_{\beta(2\ell'), j_1 \dots j_k}^{(s',c')}]_{PB} \\ &= \sum_{\ell''=c''} C_{\alpha(2\ell), \beta(2\ell')}^{\ell'' \ell''} \gamma^{(2\ell'')} [f_{cc'c''}^{\ell'' \ell''} (s, s') T_{\gamma(2\ell''), i_1 \dots i_k j_1 \dots j_k}^{(s+s'-1, c'')} + kk' C_{cc'c''}^{\ell'' \ell''} \text{Alt}(\delta_{ij_1} T_{\gamma(2\ell''), i_1 \dots i_k - i_2 \dots j_k}^{(s+s', c'')})] \end{aligned} \tag{10}$$

(see (I.13)-(I.16)). The quantities $f_{cc'c''}^{\ell'' \ell''} (s, s')$ are given in (I.16)).

The generators with $k+2s=2$ form a basis in the subsuperalgebra which is a 3D counterpart of the $SO(N)$ -superconformal algebra [3]. In particular we present as evidence the manifest expressions for the 3D counterpart of the $N=1$ Neveu-Schwarz superalgebra $PAC(osp(1/4))$ ($L_{\alpha(2\ell)}^c = T_{\alpha(2\ell)}^{(1,c)}, Q_{\alpha(2\ell)}^c = T_{\alpha(2\ell)}^{(1/2,c)} \psi$),

$$\begin{aligned} [L_{\alpha(2\ell)}^c, L_{\beta(2\ell')}^c]_{PB} &= \sum_{\ell''=c''} f_{cc'c''}^{\ell'' \ell''} (1, 1) C_{\alpha(2\ell), \beta(2\ell')}^{\ell'' \ell''} \gamma^{(2\ell'')} L_{\gamma(2\ell'')}^c, \\ [L_{\alpha(2\ell)}^c, Q_{\beta(2\ell')}^c]_{PB} &= \sum_{\ell''=c''} f_{cc'c''}^{\ell'' \ell''} (1, \frac{1}{2}) C_{\alpha(2\ell), \beta(2\ell')}^{\ell'' \ell''} \gamma^{(2\ell'')} Q_{\gamma(2\ell'')}^c, \end{aligned} \tag{11}$$

$$\{Q_{\alpha(2\ell)}^c, Q_{\beta(2\ell')}^c\}_{PB} = \sum_{\ell', c'} C_{\alpha c' c}^{2\ell' \ell} C_{\alpha(2\ell), \beta(2\ell')} \gamma^{(2\ell')} L_{\gamma(2\ell')}^c \quad (11 \text{ cont'd})$$

Here the generators

$$P_{\alpha(2)} \sim L_{\alpha(2)}^{-1}, \quad K_{\alpha(2)} \sim L_{\alpha(2)}^1, \quad M_{\alpha(2)} \sim L_{\alpha(2)}^0, \quad D \sim L^0, \quad Q_{\alpha} \sim Q_{\alpha}^{-1/2}, \quad S_{\alpha} \sim Q_{\alpha}^{1/2} \quad (12)$$

form a basis in $\text{osp}(1|4)$, which is contained as a maximal finite-dimensional subalgebra in $\text{PAC}(\text{osp}1|4)$, its analytic continuation beyond the boundaries $\ell \leq s$ ($s=1$ for $L_{\alpha(2\ell)}^c$, $s=\frac{1}{2}$ for $Q_{\alpha(2\ell)}^c$).

The higher spin extension of (11), counterpart of the Poisson-bracket higher spin extension of the Neveu-Schwarz superalgebra, is given by

$$\begin{aligned} [T_{\alpha(2\ell)}^{(s,c)}, T_{\beta(2\ell')}^{(s',c')}]_{PB} &= \sum_{\ell', c'} f_{\alpha c' c}^{2\ell' \ell} (s, s') C_{\alpha(2\ell), \beta(2\ell')} \gamma^{(2\ell')} T_{\gamma(2\ell')}^{(s+s'-1, c')}, \\ [T_{\alpha(2\ell)}^{(s,c)}, Q_{\beta(2\ell')}^{(s',c')}]_{PB} &= \sum_{\ell', c'} f_{\alpha c' c}^{2\ell' \ell} (s, s') C_{\alpha(2\ell), \beta(2\ell')} \gamma^{(2\ell')} Q_{\gamma(2\ell')}^{(s+s'-1, c')}, \\ \{Q_{\alpha(2\ell)}^{(s,c)}, Q_{\beta(2\ell')}^{(s',c')}\}_{PB} &= \sum_{\ell', c'} C_{\alpha c' c}^{2\ell' \ell} C_{\alpha(2\ell), \beta(2\ell')} \gamma^{(2\ell')} T_{\gamma(2\ell')}^{(s+s', c')}. \end{aligned} \quad (13)$$

4. Extended supersymmetric versions of $SU(\infty)$

As was shown in ref. [1], the algebra $AC(\text{so}(4,1))$ contains $SU(\infty)$, the Poisson bracket algebra on the sphere S^2 [12], as its zero-conformal weight subalgebra. It is constituted by the generators having zero Poisson brackets with the dilatation generator D . Similar $2N$ -extended supersymmetric versions of $\text{PAC}(\text{so}(3,2))$ constructed in the previous section contain extended supersymmetric versions of $SU(\infty, \infty)$ (non-compact form of $SU(\infty)$ related with $S^{1,1}$ in place of S^2) as their special subalgebras.

To describe these subsuperalgebras, let us consider the $2N$ -extended case and choose $2N$ new Grassmann variables ξ_i, η_j with the Poisson brackets

$$\{\xi_i, \eta_j\}_{PB} = \delta_{ij} \quad (14)$$

instead of the old variables ψ . Now we can introduce the following quantity:

$$S = \frac{1}{2} (\alpha_\gamma b^\gamma + \xi_i \eta^i), \quad (15)$$

which in the twistor theory is usually called a superspinality operator ((15) is its classical version exactly). It is easy to see that the condition

$$[S, f]_{PB} = 0 \quad (16)$$

for $f \in \text{osp}(2N|4)$ extracts the subsuperalgebra $SU(1, 1|N) \subset \text{osp}(2N|4)$ (we have excluded S itself) (see ref. [10]). Requiring the condition (16) to be satisfied after the analytic continuation, we arrive at the infinite-dimensional superalgebra with basis polynomials

$$T_{\alpha(2\ell), i_1 \dots i_\ell}^{j_1 \dots j_\ell} = \frac{1}{2} \sqrt{\frac{(2\ell)!}{[\ell + (q-p)/2]! [\ell - (q-p)/2]!}} \underbrace{a_{\alpha_1} \dots a_{\alpha_\ell}}_{\ell - (q-p)/2} \underbrace{b_{\alpha_1} \dots b_{\alpha_\ell}}_{\ell + (q-p)/2} \underbrace{\xi_{i_1} \dots \xi_{i_p}}_p \underbrace{\eta_{j_1} \dots \eta_{j_q}}_q (\alpha^\gamma b_\gamma)^{1 - \ell - (q-p)/2}, \quad (17)$$

where

$$\ell = 0, \frac{1}{2}, 1, \dots, \quad q, p = 0, 1, \dots, N, \quad |(q-p)/2| \leq \ell, \quad (18)$$

$|(q-p)/2|$ and ℓ are simultaneously integers or half-integers.

Supercommutation relations can be calculated straightforwardly,

$$\begin{aligned}
 [T_{\alpha(2\ell),i_1\dots i_p}^{j_1\dots j_q}, T_{\beta(2\ell'),i'_1\dots i'_p}^{j'_1\dots j'_q}]_{PB} = & \sum_{\ell'} (-1)^{qp'} C_{\alpha(2\ell),\beta(2\ell')}^{\gamma(2\ell'')} \\
 & \times [f_{(q-p)/2,(q'-p')/2,(q+q'-p-p')/2}^{\ell,\ell',\ell''} (1-(q+p)/2, 1-(q'+p')/2) T_{\gamma(2\ell''),i_1\dots i_p,i'_1\dots i'_p}^{j_1\dots j_q,j'_1\dots j'_q} \\
 & + \frac{1}{2} (-1)^{q+p'} C_{(q-p)/2,(q'-p')/2,(q+q'-p-p')/2}^{\ell,\ell',\ell''} \\
 & \times \text{Alt}(q' p \delta_{i'_p}^{j'_1} T_{\gamma(2\ell''),i_1\dots i_p,i'_1\dots i'_p}^{j_1\dots j_q,j'_2\dots j'_q} - q p' \delta_{i'_1}^{j'_q} T_{\gamma(2\ell''),i_1\dots i_p,i'_1\dots i'_p}^{j_1\dots j_q,j'_1\dots j'_q})], \tag{19}
 \end{aligned}$$

where $f_{\alpha\beta\gamma}^{\ell\ell'\ell''}(s, s')$ have been given in (I.16) and Alt means the same as in (3).

The above superalgebra as a matter of fact is an N -extended supersymmetric version of the Poisson bracket algebra on the hyperboloid $S^{1,1}$. The latter is a subalgebra and formed by the subset of the basis elements (17) without ξ and η (simply $T_{\alpha(2\ell)}$) (as it was shown in ref. [1], (I.25)). On the other hand the superalgebra (19) contains a maximal finite-dimensional subalgebra $SU(1,1|N)$ (when $2\ell + p + q = 2$ in (17)), N -extended supersymmetrization of the isometry algebra of $S^{1,1}$. Note that transformation from the usual coordinates X_μ of $S^{1,1}$ ($X_\mu X^\mu = R^2$) to our spinorial coordinates a_α, b_α as a matter of fact is a twistor-type transformation for 3D space-time (see appendix C in ref. [5])

$$x_\mu = \sigma_\mu^{\alpha(2)} a_\alpha b_\alpha \tag{20}$$

($\sigma_\mu = (I, \sigma_1, \sigma_3)$, σ_1, σ_3 are the symmetric Pauli matrices). As is known, in many cases the twistor transformation allows one to simplify considerably various non-linear structures. In the case in question it reduces the non-canonical Poisson brackets $[X_\mu, X_\nu]_{PB} = \epsilon_{\mu\nu\rho} X^\rho$ to the usual canonical ones $[a_\alpha, b_\beta]_{PB} = \epsilon_{\alpha\beta}$. Realization (17) in fact gives a generalization to supertwistors.

Now let us pass to the euclidean case. Instead of the spinorial basis the weight basis can be used with the euclidean hermitian conjugation

$$(T_{m,i_1\dots i_p}^{j_1\dots j_q})^\dagger = (-1)^{m+(q-p)/2} T_{-m,j_1\dots j_q}^{i_1\dots i_p} \tag{21}$$

and the supercommutation relations read as in (19) with the usual Clebsch-Gordan coefficients $C_{mm'm}^{\ell\ell'\ell''}$ in place of the spinorial $C_{\alpha(2\ell),\beta(2\ell')}^{\gamma(2\ell'')}$. This is an N -extended supersymmetrization of $SU(\infty)$ [12]. It contains $su(\infty)$ itself (the generators T_m^{ℓ} ($p=q=0$)) as well as the maximal finite-dimensional subalgebra $SU(2|N)$ (the generators with $2\ell + p + q = 2$).

The case of the $N=1$ superalgebra, $SU(\infty+1|\infty)$, was obtained by us also in essentially another way as the large- M limit of the finite-dimensional matrix superalgebras $SU(M+1|M)$ [13].

5. Summary

To summarize the results of the present paper, first, infinite-dimensional superalgebras generalizing 3D conformal superalgebras $osp(N|4)$ are constructed. They are 3D algebraical analogues of the Neveu-Schwarz and the $SO(N)$ -extended 2D superconformal algebras. Second, their higher spin generalizations are obtained. Third, from the $SO(2N)$ -extended 3D Virasoro-like superalgebra we have extracted a special subalgebra which can be considered as an N -extended supersymmetric version of $SU(\infty)$ (in the euclidean case).

The complete expressions for the structure constants of the above algebras are obtained. Note also that calculations of the structure constants (which are rather complicated themselves) are especially simple in the approach presented, based on the Poisson brackets for 3D supertwistors.

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